

Mass composition of cosmic rays in anomalous diffusion model: comparison with experiment

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Abstract. We calculate the energy spectra and mass composition of cosmic rays in energy region $(1 \div 10^8)$ GeV/particle under the assumption that cosmic rays propagation in the Galaxy is described by anomalous diffusion equation. Our results and comparisons with experimental data are presented.

1 Introduction

The steepening of the all-particle spectrum around $3 \cdot 10^{15}$ eV (the “knee”) discovered in 1958 (Kulikov and Khristiansen, 1958) has been the subject of numerous speculations on the propagation and acceleration mechanisms of galactic cosmic rays (see, for example, the reviews by Erlykin (1995), Kalmykov and Khristiansen (1995), Ptuskin (1997)). The usually accepted picture of cosmic ray propagation in the interstellar medium is a normal diffusion, which can be described by equation for concentration (Ginzburg and Syrovatskii, 1964; Berezhinsky et al., 1990)

$$\frac{\partial N}{\partial t} = D(E)\Delta N(\mathbf{r}, t, E) + S(\mathbf{r}, t, E), \quad (1)$$

The “knee” is not an intrinsic property of this model under the natural physical conditions $D(E) \sim E^\delta$, $S(E) \sim E^{-p}$. Because of these reasons discussions have mainly been directed to a search and a justification of breakdown mechanisms either of the diffusivity $D(E)$ or of the source density $S(E)$. However, in spite of considerable theoretical and experimental efforts, a model, which can explain

1. different values of spectral exponent of protons and other nuclei,
2. mass composition variations at $E \sim 10^2 \div 10^5$ GeV/nucleon,
3. the steepening of the all-particle spectrum

was not developed.

Recently, in our papers (Lagutin et al., 2000, 2001b,c) new view of the “knee” problem was presented. It has been shown that the “knee” in the primary cosmic rays spectrum is due to fractal structure of the interstellar medium, that is another regime of particles diffusion in the Galaxy.

In this paper we consider the propagation of galactic cosmic rays in the fractal interstellar medium taking into account that a particle can spend long time in a trap. We demonstrate the main results of this new model in a wide energy region $E \sim (1 \div 10^8)$ GeV/nucleon.

2 Model

Based on the results (Lagutin and Uchaikin, 2001) the cosmic ray propagation in the fractal interstellar medium is described by fractional diffusion equation. Without energy losses and nuclear interactions, the equation for concentration of the cosmic rays with energy E generated by sources $S(\mathbf{r}, t, E)$ has the form

$$\frac{\partial N}{\partial t} = -D(E, \alpha, \beta) D_{0+}^{1-\beta} (-\Delta)^{\alpha/2} N(\mathbf{r}, t, E) + S(\mathbf{r}, t, E) \quad (2)$$

where $D(E, \alpha, \beta)$ is the anomalous diffusivity, α and β are determined by the fractal structure of the medium and by the trapping mechanism, correspondingly (see Lagutin and Uchaikin (2001)). D_{0+}^μ denotes the Riemann-Liouville fractional derivative, $(-\Delta)^{\alpha/2}$ — the fractional Laplacian (Samko et al., 1987).

In the case of point impulse source with inverse power spectrum, relating to supernova bursts

$$S(\mathbf{r}, t, E) = S_0 E^{-p} \delta(\mathbf{r}) \Theta(T - t) \Theta(t), \quad \Theta(\tau) = \begin{cases} 1, & \tau > 0, \\ 0, & \tau < 0, \end{cases}$$

the solution of equation (2), found in (Lagutin et al., 2001b)

is of the form

$$N(\mathbf{r}, t, E) = \frac{S'_0 E^{-p}}{D(E, \alpha, \beta)^{3/\alpha}} \int_{\max[0, t-T]}^t \tau^{-3\beta/\alpha} \times \Psi_3^{(\alpha, \beta)} \left(|\mathbf{r}| (D(E, \alpha, \beta) \tau^\beta)^{-1/\alpha} \right) d\tau, \quad (3)$$

where the scaling function $\Psi_3^{(\alpha, \beta)}(r)$,

$$\Psi_3^{(\alpha, \beta)}(r) = \int_0^\infty q_3^{(\alpha)}(r \tau^\beta) q_1^{(\beta, 1)}(\tau) \tau^{3\beta/\alpha} d\tau, \quad (4)$$

is determined by three-dimensional spherically-symmetrical stable distribution $q_3^{(\alpha)}(r)$ ($\alpha \leq 2$) and one-sided stable distribution $q_1^{(\beta, 1)}(t)$ with characteristic exponent β (Zolotarev, 1983; Uchaikin and Zolotarev, 1999).

The anomalous diffusivity $D(E, \alpha, \beta)$ is determined by the constants A and B in the asymptotic behaviour for “Lévy flights” (A) and “Lévy waiting time” (B) distributions:

$$D(E, \alpha, \beta) \propto A(E, \alpha)/B(E, \beta).$$

Taking into account that both the free path and the probability to stay in trap during the time interval t for particle with charge Z and mass number A depend on particle magnetic rigidity R , we accept $D = (v/c) D_0(\alpha, \beta) R^\delta$.

Using the representation $N = N_0 E^{-\eta}$ and the property $d\Psi_m^{(\alpha, \beta)}/dr = -2\pi r \Psi_{m+2}^{(\alpha, \beta)}$ of the scaling function (Uchaikin and Zolotarev, 1999), one can easily find the spectral exponent η for observed particles:

$$\eta = p + \frac{\delta}{\alpha} \Xi, \quad (5)$$

where

$$\Xi = 3 - \frac{2\pi r^2}{D(E, \alpha, \beta)^{2/\alpha}} \times \frac{\int_{\max[0, t-T]}^t \tau^{-5\beta/\alpha} \Psi_5^{(\alpha, \beta)} \left(|\mathbf{r}| (D(E, \alpha, \beta) \tau^\beta)^{-1/\alpha} \right) d\tau}{\int_{\max[0, t-T]}^t \tau^{-3\beta/\alpha} \Psi_3^{(\alpha, \beta)} \left(|\mathbf{r}| (D(E, \alpha, \beta) \tau^\beta)^{-1/\alpha} \right) d\tau}. \quad (6)$$

Let E_0 be a solution of the equation $\Xi(E) = 0$. One can see from (5)-(6) that at $E = E_0$ the spectral exponent for observed particles η is equal to spectral exponent for particles generated by the source: $\eta(E_0) = p$. Since the exponent $\eta_{E \ll E_0} = p - \delta$ is less than p at $E \ll E_0$, but the exponent $\eta_{E \gg E_0} = p + \delta/\beta > p$ at $E \gg E_0$, E_0 can be called the “knee” energy.

From experimental values of $\eta_{E \ll E_0}$ and $\eta_{E \gg E_0}$ one can derive the main parameters of the model (p, δ) versus the spectral exponent (β) of “the Lévy waiting time”:

$$\delta = (\eta_{E \gg E_0} - \eta_{E \ll E_0}) \frac{\beta}{1 + \beta}, \quad p = \eta_{E \ll E_0} + \delta.$$

To evaluate the parameter β we have used the results presented in the paper (Cadavid et al., 1999), where an anomalous diffusion of solar magnetic elements have been investigated. The authors have shown that the trapping time distribution asymptotically takes the form of a Lévy distribution with spectral exponent $\beta \approx 0.8$.

Assuming that a trapping mechanism is characterized by a kind of self-similarity, one can expect the same value for β in the scales under the consideration. By this reason the value $\beta = 0.8$ is used in our calculations.

Thus, taking $\eta_{E \ll E_0} \sim 2.63$ and $\eta_{E \gg E_0} \sim 3.24$ we finally obtain $p \approx 2.9$, $\delta \approx 0.27$.

To evaluate the next important parameter — anomalous diffusivity $D_0(\alpha, \beta)$, we have used the experimental data on the particle anisotropy in the energy region $10^3 \div 10^4$ GeV/particle in the framework of the scheme proposed by Osborn et al. (1976) and Dorman et al. (1985). For example, we find $D_0 \approx (1 \div 4) \cdot 10^{-3} \text{ pc}^{1.7} \text{ y}^{-0.8}$ in the case $\alpha = 1.7$, $\beta = 0.8$ for near sources Monogem ($r \sim 300 \text{ pc}$, $t \sim 10^5 \text{ y}$), Geminga ($r \sim 300 \text{ pc}$, $t \sim 3 \cdot 10^5 \text{ y}$), Loop I-IV ($r \sim (100 \div 200) \text{ pc}$, $t \sim (2 \div 4) \cdot 10^5 \text{ y}$).

In the model under consideration only one parameter α ($1 < \alpha < 2$) connected with the fractal structure of the interstellar medium is found by fit. Extensive calculations of cosmic-ray spectra show that the best fit of experimental data may be get at $\alpha \approx 1.7$.

3 Spectra and mass composition

The differential flux J_i of the particles of type i due to all sources of Galaxy may be separated into two components

$$J = J_L(r \leq 1 \text{ kpc}) + J_G(r > 1 \text{ kpc}). \quad (7)$$

The first component (L) in (7) describes the contribution of the nearby sources (at distance $r \leq 1 \text{ kpc}$) to observed flux J_i . The second component (G) is the contribution of the distant sources ($r > 1 \text{ kpc}$) to J_i . The similar separation is frequently used in the studies of cosmic rays (see, for instance, Atoyan et al. (1995)).

The list of nearby sources including 16 supernova remnants (Nichimura et al., 1979, 1995; Lozinskaya, 1986) is used to calculate the L-component. The distant sources are supposed to be distributed uniformly both in space and time. In this case $J_G(r > 1 \text{ kpc}) \sim E^{-p-\delta/\beta}$ (see Lagutin et al. (2001a)).

Based on this result and (3), we present the differential flux in the form:

$$J_i(E) = \frac{v_i}{4\pi} \sum_{i, (r \leq 1 \text{ kpc})} N(\mathbf{r}_i, t_i, E) + v_i C_{0i} E^{-p-\delta/\beta}. \quad (8)$$

It is clear from physical point of view that the bulk of observed cosmic rays with energy $10^8 \div 10^{10} \text{ eV}$ forms by numerous distant sources. It means that the observed flux at least of protons and He in this energy region must be described by second term in (8). The first term in (8) defines the

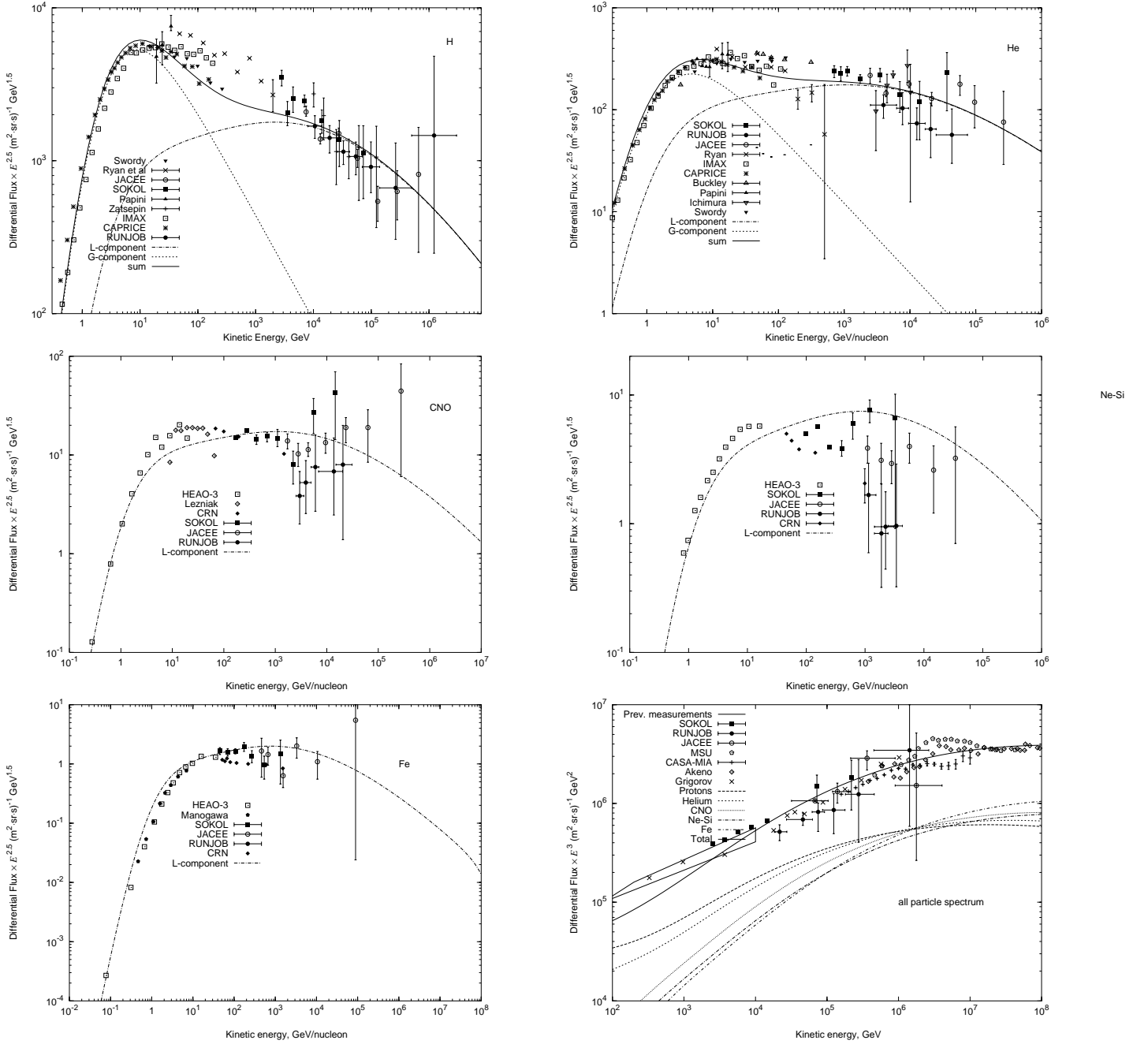


Fig. 1. Comparison of our calculations of spectra with experimental data:

Grigirov — Grigorov et al. (1970),
 Ryan — Ryan et al. (1972),
 Minagawa — Minagawa (1981),
 CRN — Grunsfeld et al. (1988),
 HEAO-3 — Engelmann et al. (1990),
 SOKOL — Ivanenko et al. (1990, 1993),
 MSU — Fomin et al. (1991),
 Swordy — Muller et al. (1991),
 Ichimura — Ichimura et al. (1993),
 Zaitsepin — Zaitsepin et al. (1993),
 Buckley — Buckley et al. (1994),
 Papini, Lezniak — Wiebel et al. (1994),
 JACEE — Asakimory et al. (1998),
 CAPRICE — Boezio et al. (1999),
 CASA-MIA — Glasmacher et al. (1999),
 AKENO — Youshida et al. (2001),
 RUNJOB — Apanasenko et al. (2001).

E,GeV	H	He	CNO	Ne-Si	Fe	$\langle \ln A \rangle$	$\langle A \rangle$
1E+2	0.53	0.32	0.08	0.05	0.02	0.90	5.36
3E+2	0.45	0.30	0.12	0.08	0.06	1.20	8.41
1E+3	0.40	0.29	0.14	0.10	0.08	1.40	10.38
3E+3	0.36	0.28	0.16	0.11	0.10	1.53	11.65
1E+4	0.33	0.27	0.17	0.12	0.11	1.65	12.91
3E+4	0.30	0.26	0.18	0.14	0.13	1.79	14.37
1E+5	0.26	0.24	0.19	0.15	0.15	1.93	15.95
3E+5	0.24	0.23	0.20	0.16	0.17	2.06	17.54
1E+6	0.21	0.21	0.20	0.17	0.20	2.17	19.06
3E+6	0.19	0.20	0.21	0.18	0.22	2.28	20.44
1E+7	0.18	0.19	0.21	0.19	0.24	2.37	21.68
3E+7	0.16	0.18	0.21	0.19	0.26	2.45	22.76
1E+8	0.15	0.17	0.21	0.20	0.27	2.51	23.70
1E+8	0.15	0.17	0.21	0.20	0.27	2.52	23.87

Table 1. Mass composition of cosmic rays in anomalous diffusion model

spectrum in the high energy region and, as has been shown above, provides the “knee”.

We use the spherically symmetric force model of Axford and Gleeson (1968) to describe the solar modulation. The influence of solar modulation on the particle flux is

$$J_{mod}(T) = \frac{T^2 + 2m_p c^2 T}{(T + \Phi)^2 + 2m_p c^2 (T + \Phi)} J_{ISM}(T + \Phi),$$

where T is the kinetic energy per nucleon, m_p is the mass of a proton and J_{ISM} is the interstellar flux (8). The potential energy Φ , describing the average energy loss of particle from interstellar space to 1 AU, is determined by solar modulation parameter ϕ : $\Phi = \phi Z/A$. $\phi = 750 MV$ is accepted in this paper (see Boezio et al. (1999); Menn et al. (2000)).

The results of our calculation are presented in Fig. 1, and Table 1.

4 Conclusion

We have considered the propagation of galactic cosmic rays in the fractal interstellar medium. The energy spectra of nuclei (H, He, CNO, Ne-Si, Fe) and mass composition have been calculated in energy region ($1 \div 10^8$) GeV/particle under the natural physical conditions $D(E) \sim E^\delta$, $S(E) \sim E^{-p}$. We have shown that the model can explain the different values of spectral exponent of protons and other nuclei, mass composition variations at $E \sim 10^2 \div 10^5$ GeV/nucleon, the steepening of the all-particle spectrum.

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